Advanced Encryption for the Sharing of Sensitive Data

PHD Defense

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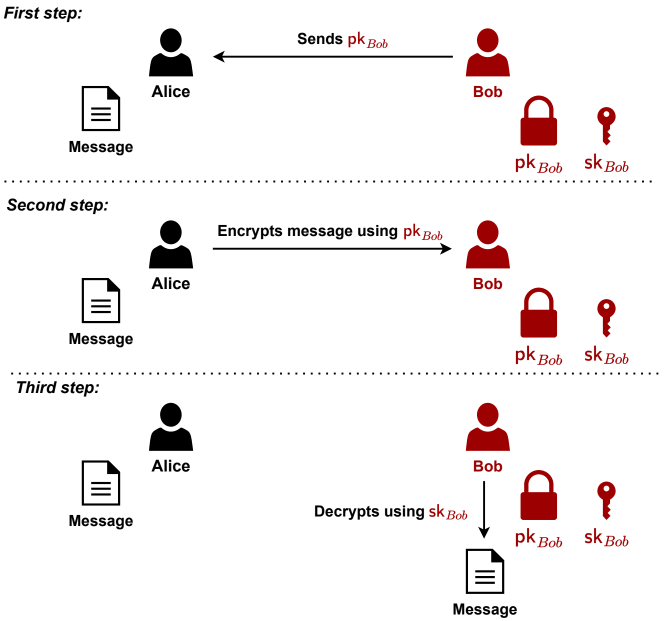
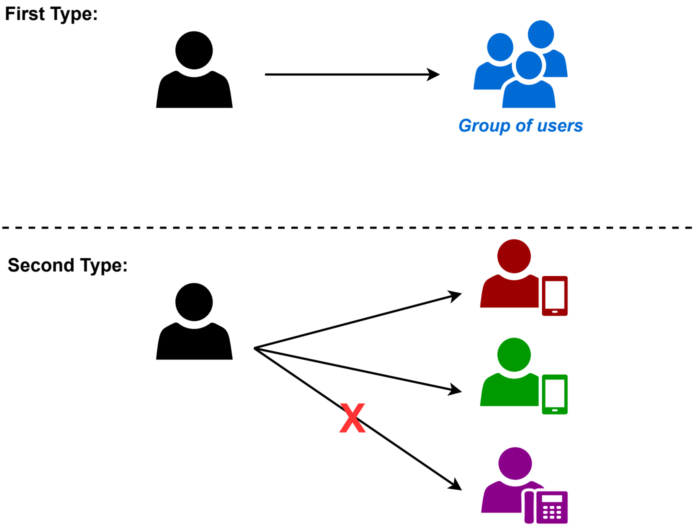
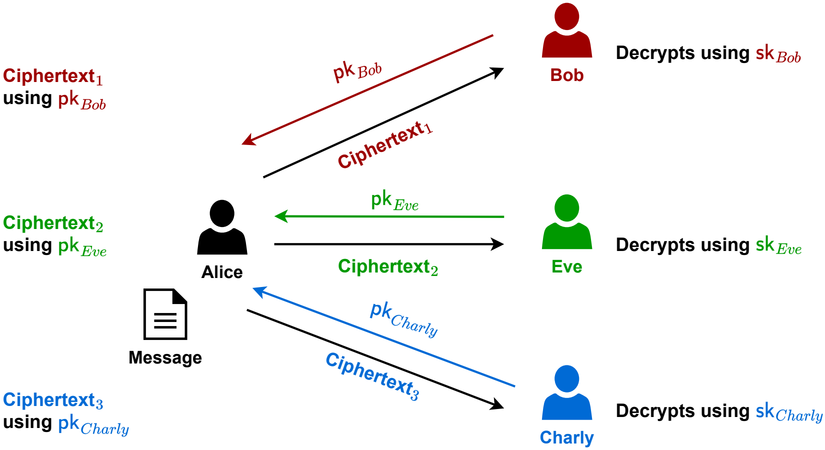
Asymmetric Cryptography



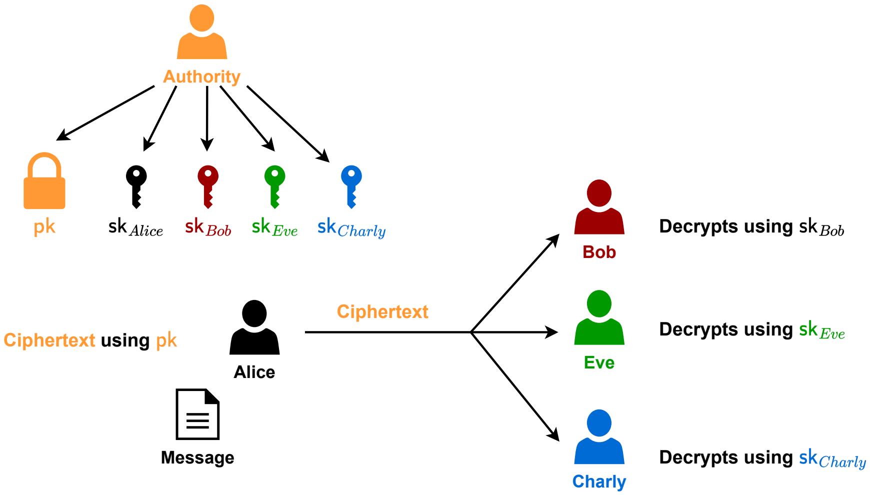
Two Types of Sharing



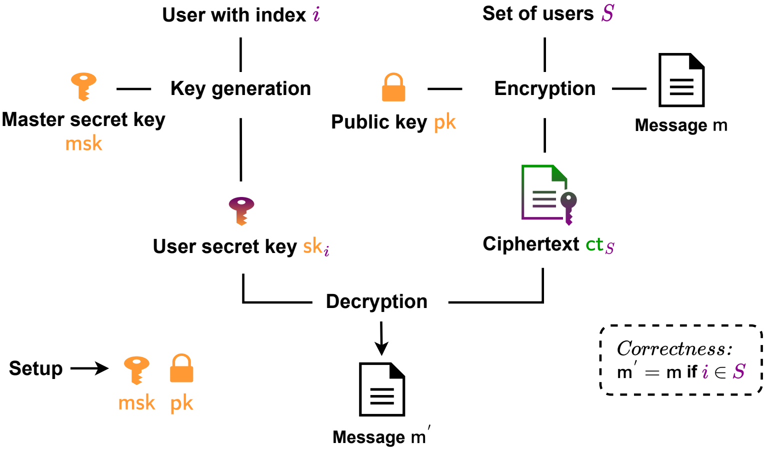
Sharing to Several Persons: Trivial Way



Sharing to Several Persons: Efficient Way

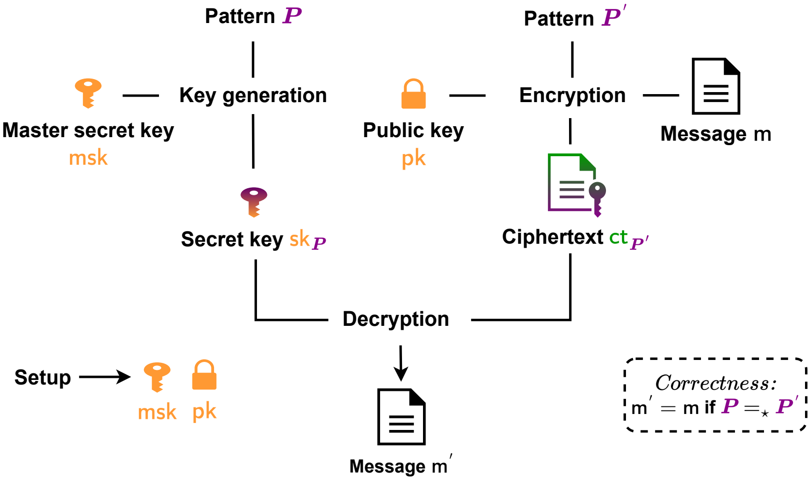


Advanced Encryption Scheme For Sharing to a Group of Users



Broadcast Encryption scheme

First Tool: Identity-Based Encryption with Wildcards



Our Contributions (1)

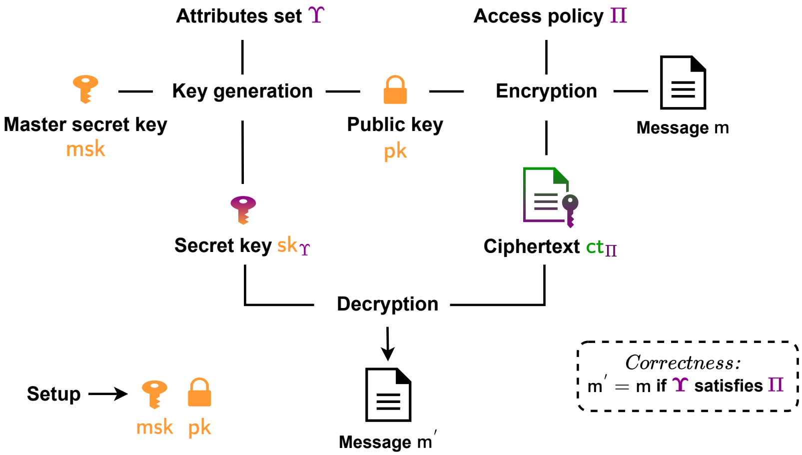
Main Contributions

- **Generic construction of Broadcast Encryption scheme from Identity-Based Encryption with Wildcards**
- New pairing-based Broadcast Encryption scheme with *constant size ciphertext*

Auxiliary Contribution

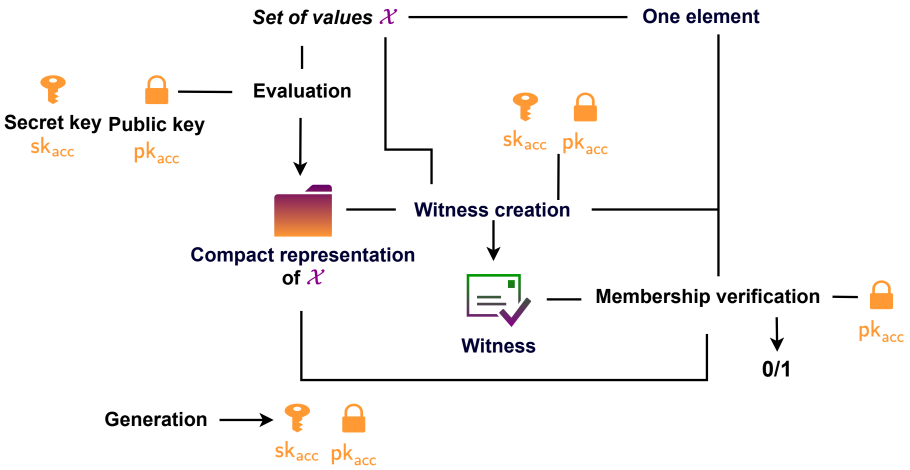
- New pairing-based Identity-Based Encryption with Wildcards scheme, with *constant size ciphertext*

Advanced Encryption Scheme For Sharing to a Group With Common Attributes



Ciphertext Policy Attribute-Based Encryption scheme

Second Tool: Cryptographic Accumulators



Our Contributions (2)

Main Contribution

- **New pairing-based Ciphertext Policy Attribute-Based Encryption with both constant size ciphertext and secret keys based on Cryptographic Accumulators**

Auxiliary Contributions

- **Introducing a new type of Cryptographic Accumulators: dually computable accumulators**
- **First dually computable accumulator scheme, based on pairings**

Going Further: Our Other Contributions

In Submission

- Main contribution:
  - An Attribute-Based Encryption scheme from Identity-Based Encryption with Wildcards, protecting privacy of *both* access policies and attributes
- Auxiliary contributions:
  - Introducing a new functionality for Identity-Based Encryption with Wildcards scheme: *privacy-preserving key generation*
  - Pairing-based privacy-preserving key generation Identity-Based Encryption with Wildcards scheme

Cryptographic Accumulators Systematization of Knowledge (*In submission*)

- New security property, *unforgeability of private evaluation*
- Discussions on applications and properties of accumulators

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Broadcast Encryption

Broadcast Encryption (BE) [FN94]

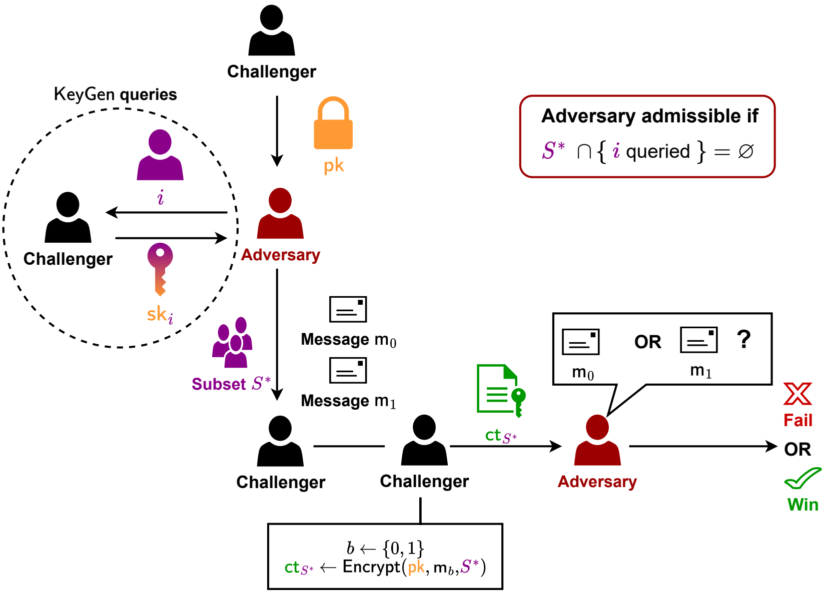
- $\mathsf{Setup}(1^\lambda, N) \to (\mathsf{pk}, \mathsf{msk})$
- $\mathsf{Encrypt}(\mathsf{pk}, \mathsf{m}, S) \to \mathsf{ct}_S$
- $\mathsf{KeyGen}(\mathsf{msk}, i) \to \mathsf{sk}_i$ for $i = 1, \cdots, N$
- $\mathsf{Decrypt}(\mathsf{sk}_i, \mathsf{ct}_S, S) \to \mathsf{m}'$

Correctness:

For all $\lambda, N \in \mathbb{N}$, for $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda, N)$ **honestly generated** and for all index and subset i, S such that $i \in S$:

$$\mathsf{Decrypt}(\mathsf{KeyGen}(\mathsf{msk}, i), \mathsf{Encrypt}(\mathsf{pk}, \mathsf{m}, S), S) = \mathsf{m}$$

Broadcast Encryption Security: Indistinguishability



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Patterns

Patterns [ACD+06, KLLO18]

- Pattern $\boldsymbol{P} = (P_1, \cdots, P_L) \in \mathcal{U}^L$, where
  - $\mathcal{U}$: set with a special wildcard symbol "$\star$",
  - $L \in \mathbb{N}$

- $\boldsymbol{P}^{'} = (P_1^{'}, \cdots, P_L^{'})$ and $\boldsymbol{P} = (P_1, \cdots, P_L)$:
  - $\boldsymbol{P}$ **belongs** to $\boldsymbol{P}^{'}$, denoted $\boldsymbol{P} \in_\star \boldsymbol{P}^{'}$, iff $\forall i \in \{1, \cdots, L\}$, $(P_i^{'} = P_i) \vee (P_i^{'} = \star)$
  - $\boldsymbol{P}$ **matches** $\boldsymbol{P}^{'}$, denoted $\boldsymbol{P} =_\star \boldsymbol{P}^{'}$, iff $\forall i \in \{1, \cdots, L\}$, $(P_i^{'} = P_i) \vee (P_i = \star) \vee (P_i^{'} = \star)$

Patterns: Example

$$\mathcal{U} = \{0, 1, \star\}$$

$$P = 0\ 1\ 1\ \star\ 0$$
$$P' = \star\ 1\ 1\ 0\ \star$$

$$P =_\star P'$$

$$P = 0\ 1\ 1\ \star\ 0$$
$$P' = \star\ 0\ 1\ 0\ \star$$

$$P \neq_\star P'$$

$$P = 0\ 1\ 1\ 0\ 0$$
$$P' = \star\ 1\ 1\ 0\ \star$$

$$P \in_\star P'$$

$$P = 0\ 1\ 1\ 1\ 0$$
$$P' = \star\ 0\ 1\ 0\ \star$$

$$P \notin_\star P'$$

Equal to $\star$

Equals

Differents

Identity-Based Encryption with Wildcards

Identity-Based Encryption with Wildcards (WIBE) [ACD+06]

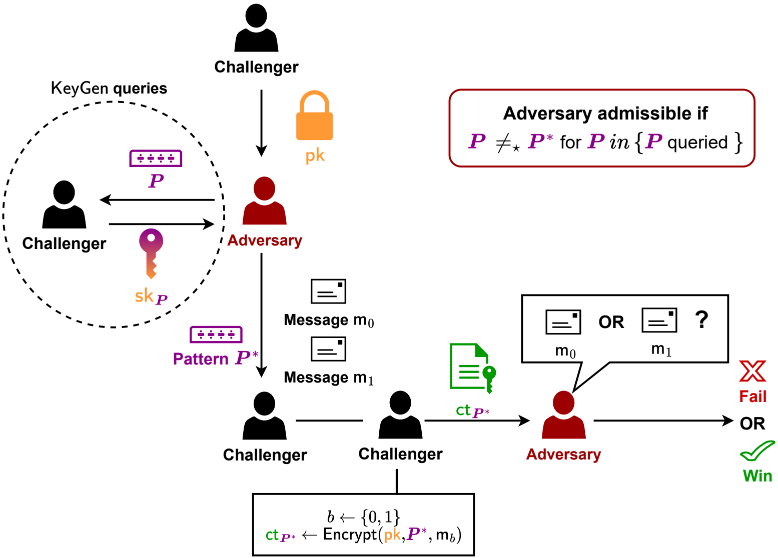
- $\mathsf{Setup}(1^\lambda, L) \to (\mathsf{pk}, \mathsf{msk})$
- $\mathsf{KeyGen}(\mathsf{msk}, \boldsymbol{P}) \to \mathsf{sk}_{\boldsymbol{P}}$
- $\mathsf{Encrypt}(\mathsf{pk}, \boldsymbol{P}', \mathsf{m}) \to \mathsf{ct}_{\boldsymbol{P}'}$
- $\mathsf{Decrypt}(\mathsf{sk}_{\boldsymbol{P}}, \boldsymbol{P}, \mathsf{ct}_{\boldsymbol{P}'}, \boldsymbol{P}') \to \mathsf{m}'$

Correctness:

For all $\lambda, L \in \mathbb{N}$, for $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda, L)$ **honestly generated** and for all patterns $\boldsymbol{P}, \boldsymbol{P}'$ such that $\boldsymbol{P} =_\star \boldsymbol{P}'$:

$$\mathsf{Decrypt}(\mathsf{KeyGen}(\mathsf{msk}, \boldsymbol{P}), \boldsymbol{P}, \mathsf{Encrypt}(\mathsf{pk}, \boldsymbol{P}', \mathsf{m}), \boldsymbol{P}') = \mathsf{m}$$

WIBE Security: Indistinguishability



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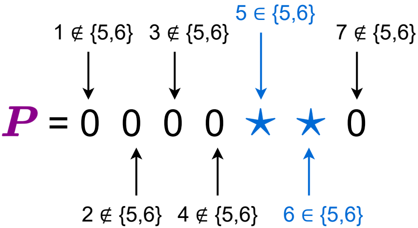
Building BE From WIBE

- Any subset $S \subseteq [N]$ can be represented as a pattern $\boldsymbol{P} \in \{0, \star\}^N$: for $j \in [1, N]$,
  - $P_j = \star$ if $j \in S$
  - $P_j = 0$ otherwise

Example: for $N = 7$

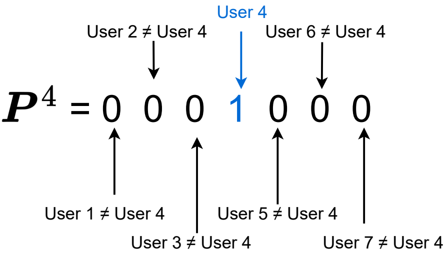


Creating a pattern in {0,★}^{7} representing the set {5,6}

Building BE From WIBE

- Any user identity $i \in [N]$ can be represented as a pattern $\boldsymbol{P^i} \in \{0,1\}^N$: for $j \in [1, N]$,
  - $P^i_j = 1$ if $j = i$
  - $P^i_j = 0$ otherwise

Example: for $N = 7$



Creating a pattern in $\{0,1\}^7$ representing identity of User 4

Building BE From WIBE

- $i \in S \iff \boldsymbol{P}^i \in_\star \boldsymbol{P}$

Example: $N = 7$, $\boldsymbol{P}$ for subset $\{5, 6\}$

$$\begin{array}{rl} \boldsymbol{P} = & (0\ 0\ 0\ 0\ \star\ \star\ 0) \\ \boldsymbol{P}^4 = & (0\ 0\ 0\ 1\ 0\ 0\ 0) \end{array} \qquad \begin{array}{rl} \boldsymbol{P} = & (0\ 0\ 0\ 0\ \star\ \star\ 0) \\ \boldsymbol{P}^6 = & (0\ 0\ 0\ 0\ 0\ 1\ 0) \end{array}$$

When User 4 tries to decrypt: $\boldsymbol{P}^4 \notin_\star \boldsymbol{P}$

When User 6 tries to decrypt: $\boldsymbol{P}^6 \in_\star \boldsymbol{P}$

Generic Construction

Ciphertext pattern space: $\{0, \star\}^N$, Key pattern space: $\{0, 1\}^N$

Broadcast Encryption from WIBE

- $\mathsf{Setup}(1^\lambda, N) = \mathsf{WIBE.Setup}(1^\lambda, N) \rightarrow (\mathsf{pk}, \mathsf{msk})$
- $\mathsf{KeyGen}(\mathsf{msk}, i \in [N]) = \mathsf{WIBE.KeyGen}(\mathsf{msk}, \boldsymbol{P}^i) \rightarrow \mathsf{sk}_{\boldsymbol{P}^i}$ for $\boldsymbol{P}^i \in \{0, 1\}^N$ as above
- $\mathsf{Encrypt}(\mathsf{pk}, S, \mathsf{m}) = \mathsf{WIBE.Encrypt}(\mathsf{pk}, \boldsymbol{P}, \mathsf{m}) \rightarrow \mathsf{ct}_{\boldsymbol{P}}$, for $\boldsymbol{P}$ in $\{0, \star\}^N$ as above
- $\mathsf{Decrypt}(\mathsf{sk}_i, \mathsf{ct}_{\boldsymbol{P}}, S) = \mathsf{WIBE.Decrypt}(\mathsf{sk}_{\boldsymbol{P}^i}, \boldsymbol{P}^i, \mathsf{ct}_{\boldsymbol{P}}, \boldsymbol{P}) \rightarrow \mathsf{m}'$

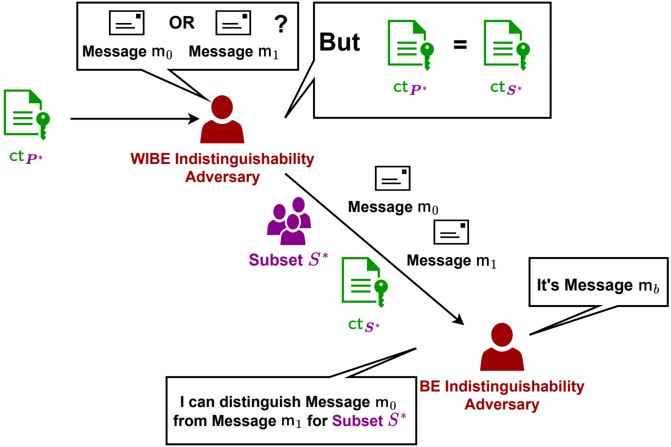
Correctness:

Correctness of the obtained BE comes from correctness of the underlying WIBE

Security

Theorem

If WIBE satisfies indistinguishability security, the obtained BE satisfies indistinguishability security.



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Other Main Contributions

- Generic construction of Augmented Broadcast Encryption scheme, a variant of Broadcast Encryption scheme, from Identity-Based Encryption with Wildcards
- First (pairing-based) Augmented Broadcast Encryption scheme secure in the *standard model*

Other Auxiliary Contributions

- New security property for Identity-Based Encryption with Wildcards: *pattern-hiding*
- First (pairing-based) Identity-Based Encryption with Wildcards scheme satisfying *pattern-hiding security*

All results are in an article accepted at CANS 2022

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Attribute-Based Encryption [SW05]

Ciphertext Policy Attribute-Based Encryption (CP-ABE)

- $\mathsf{Setup}(1^\lambda) \to (\mathsf{pk}, \mathsf{msk})$
- $\mathsf{KeyGen}(\mathsf{msk}, \mathsf{pk}, \Upsilon) \to \mathsf{sk}_\Upsilon$
- $\mathsf{Encrypt}(\mathsf{pk}, \Pi, \mathsf{m}) \to \mathsf{ct}_\Pi$
- $\mathsf{Decrypt}(\mathsf{sk}_\Upsilon, \Upsilon, \mathsf{ct}_\Pi, \Pi) \to \mathsf{m}'$

Key Policy Attribute-Based Encryption (KP-ABE)

Similar to CP-ABE except that attributes and policies are swapped in KeyGen and Encrypt

CP-ABE Properties

Correctness:

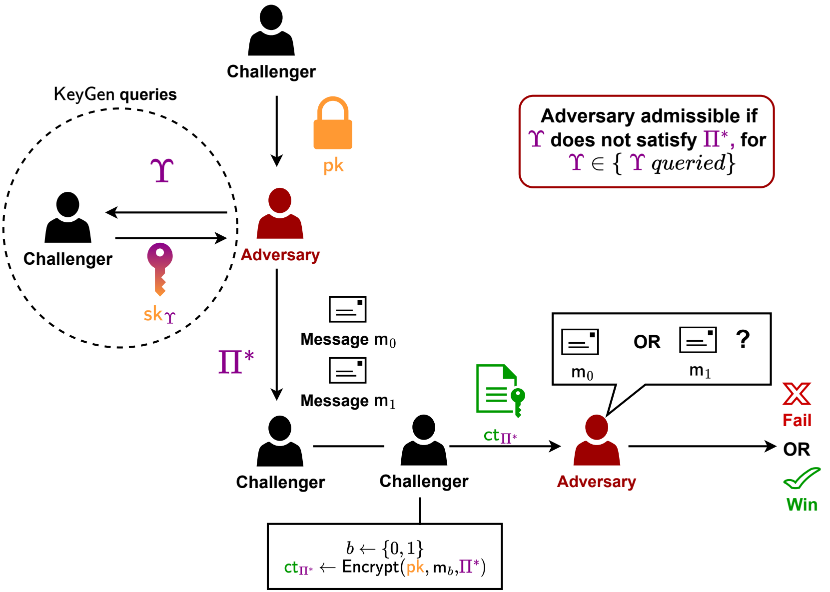
For all $\lambda \in \mathbb{N}$, for $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ **honestly generated**, and all Υ, Π such that Υ satisfies Π:

$$\mathsf{Decrypt}(\mathsf{KeyGen}(\mathsf{msk}, \mathsf{pk}, \Upsilon), \Upsilon, \mathsf{Encrypt}(\mathsf{pk}, \Pi, \mathsf{m}), \Pi) = \mathsf{m}$$

Bounded:

Number of attributes in the scheme is **bounded** by $q \in \mathbb{N}$

Attribute Based Encryption Security: Indistinguishability



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Cryptographic Accumulators [Bd94]

Asymmetric Cryptographic Accumulator

- $\mathsf{Gen}(1^\lambda) \to (\mathsf{pk_{acc}}, \mathsf{sk_{acc}})$
- $\mathsf{Eval}((\mathsf{sk_{acc}},)\mathsf{pk_{acc}}, \mathcal{X}) \to \mathsf{acc}_\mathcal{X}$
- $\mathsf{WitCreate}((\mathsf{sk_{acc}},)\mathsf{pk_{acc}}, \mathsf{acc}_\mathcal{X}, \mathcal{X}, y) \to \mathsf{wit}_y$
- $\mathsf{Verify}(\mathsf{pk_{acc}}, \mathsf{acc}_\mathcal{X}, \mathsf{wit}_y, y) \to 0/1$

Symmetric Cryptographic Accumulator

- $\mathsf{Gen}(1^\lambda) \to (\mathsf{pk_{acc}}, \mathsf{sk_{acc}})$
- $\mathsf{Eval}((\mathsf{sk_{acc}},)\mathsf{pk_{acc}}, \mathcal{X}) \to \mathsf{acc}_\mathcal{X}$
- $\mathsf{Verify}(\mathsf{pk_{acc}}, \mathsf{acc}_\mathcal{X}, y) \to 0/1$

Asymmetric Accumulators: Properties and Requirements

Correctness:

For all $\lambda \in \mathbb{N}$, for $(\mathsf{pk_{acc}}, \mathsf{sk_{acc}}) \leftarrow \mathsf{Setup}(1^\lambda)$ **honestly generated**, for all $y \in \mathcal{X}$ and $\mathsf{acc}_{\mathcal{X}} \leftarrow \mathsf{Eval}((\mathsf{sk_{acc}},)\mathsf{pk_{acc}}, \mathcal{X})$:

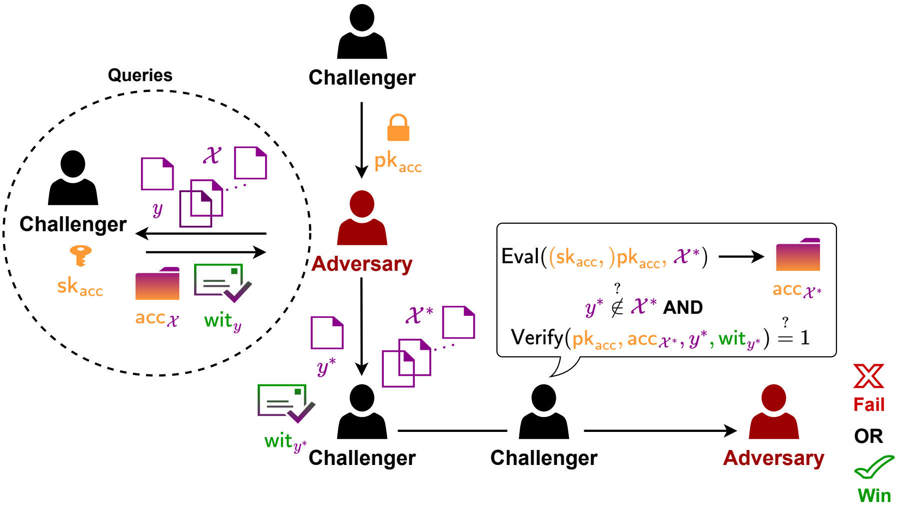
$$\mathsf{Verify}(\mathsf{pk_{acc}}, \mathsf{acc}_{\mathcal{X}}, \mathsf{WitCreate}((\mathsf{sk_{acc}},)\mathsf{pk_{acc}}, \mathsf{acc}_{\mathcal{X}}, \mathcal{X}, y), y) = 1$$

Bounded:

For all $\mathcal{X}$, $|\mathcal{X}| \leq q$, where $q \in \mathbb{N}$ is a **bound** given as input of Gen.

Sizes requirements: $|\mathsf{acc}_{\mathcal{X}}|$ and $|\mathsf{wit}_y|$ are small

Accumulators Security: Collision Resistance



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Dually Computable Cryptographic Accumulators

Dually Computable Cryptographic Accumulators

- $\mathsf{Gen}(1^\lambda) \rightarrow (\mathsf{pk_{acc}}, \mathsf{sk_{acc}})$
- $\mathsf{Eval}(\mathsf{sk_{acc}}, \mathcal{X}) \rightarrow \mathsf{acc}_{\mathcal{X}}$ Private Evaluation
- $\mathsf{WitCreate}(\mathsf{pk_{acc}}, \mathsf{acc}_{\mathcal{X}}, \mathcal{X}, y) \rightarrow \mathsf{wit}_y$ Public Witness Generation
- $\mathsf{Verify}(\mathsf{pk_{acc}}, \mathsf{acc}_{\mathcal{X}}, \mathsf{wit}_y, y) \rightarrow 0/1$

Two additional algorithms

- $\mathsf{PublicEval}(\mathsf{pk_{acc}}, \mathcal{X}) \rightarrow \mathsf{accp}_{\mathcal{X}}$
- $\mathsf{PublicVerify}(\mathsf{pk_{acc}}, \mathsf{accp}_{\mathcal{X}}, \mathsf{wit}_y, y) \rightarrow 0/1$

Dually Computable Cryptographic Accumulators
Correctness

Correctness:

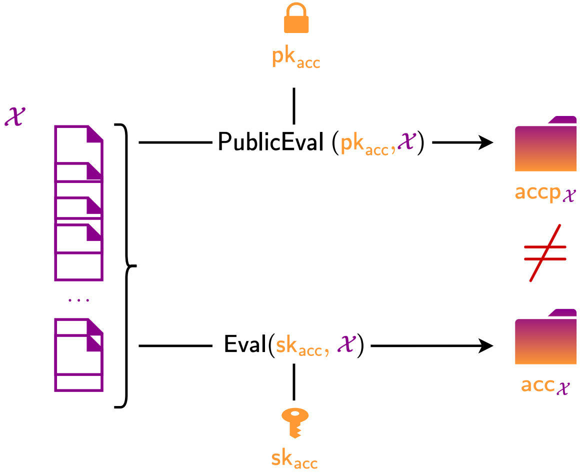
For all $\lambda \in \mathbb{N}$, for $(\mathsf{pk_{acc}}, \mathsf{sk_{acc}}) \leftarrow \mathsf{Setup}(1^\lambda)$ **honestly generated**, for all $y \in \mathcal{X}$, $\mathsf{acc}_{\mathcal{X}} \leftarrow \mathsf{Eval}(\mathsf{sk_{acc}}\mathcal{X})$ and $\mathsf{accp}_{\mathcal{X}} \leftarrow \mathsf{PublicEval}(\mathsf{pk_{acc}}, \mathcal{X})$:

$$\mathsf{Verify}(\mathsf{pk_{acc}}, \mathsf{acc}_{\mathcal{X}}, \mathsf{WitCreate}(\mathsf{pk_{acc}}, \mathsf{acc}_{\mathcal{X}}, \mathcal{X}, y), y) = 1$$

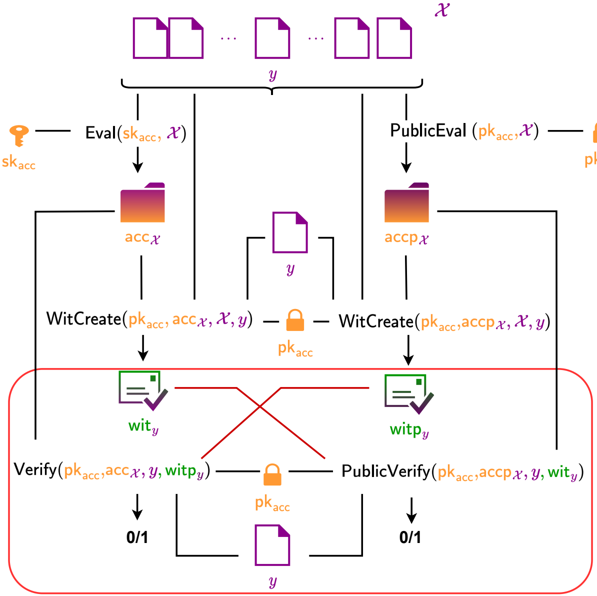
and

$$\mathsf{PublicVerify}(\mathsf{pk_{acc}}, \mathsf{accp}_{\mathcal{X}}, \mathsf{WitCreate}(\mathsf{pk_{acc}}, \mathsf{accp}_{\mathcal{X}}, \mathcal{X}, y), y) = 1$$

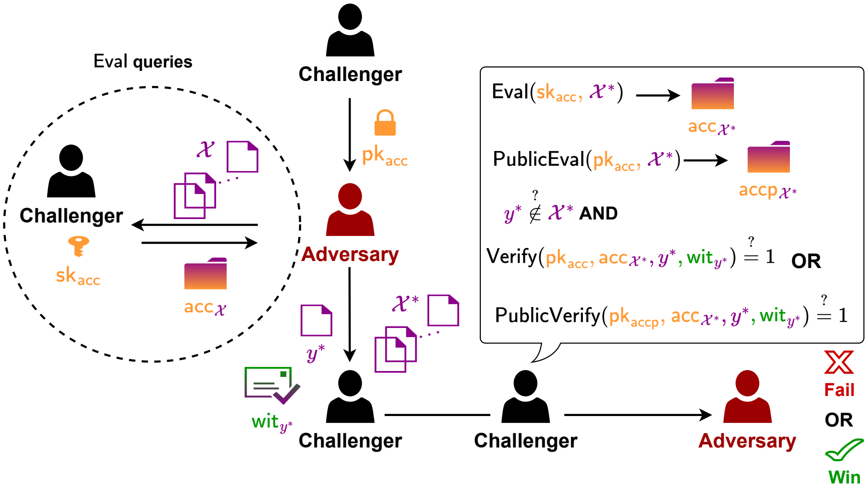
Distinguishability



Correctness of Duality



Dually Computable Accumulators Security : Dual Collision Resistance



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CP-ABE From Dually Computable Cryptographic Accumulators

Main Idea

- Use Eval on Υ and set $\mathsf{sk}_\Upsilon = \mathsf{acc}_\Upsilon$
- Use PublicEval on Π, randomize accp_Π to get accp'_Π and set $\mathsf{ct}_\Pi = \mathsf{m} \oplus \mathsf{accp}'_\Pi$
- To decrypt, compute accp'_Π

Security and Correctness

- *Protection against unauthorized decryption:* accp'_Π computable only if $\mathsf{acc}_\Upsilon \cap \mathsf{accp}_\Pi \neq \varnothing$
- *Correctness:* $\mathsf{acc}_\Upsilon \cap \mathsf{accp}_\Pi \neq \varnothing \Longleftrightarrow \Upsilon$ satisfies Π

Accumulators Over Access Policies

Access Policies: *disjunctions of conjunctions*

Our Idea

- $\mathcal{H}$: {set of attributes} $\rightarrow$ accumulator space, hash function
- For the access policy:
  - run $\mathcal{H}$ on *each set* representing a *conjunction* of Π
  - add the obtained element to a set $\mathcal{Y}$
  - run PublicEval on $\mathcal{Y}$

An Example

- $\Pi = (a_1 \wedge a_3) \vee (a_2 \wedge a_4)$
- $\mathcal{Y} = \{\mathcal{H}(\{a_1, a_3\}), \mathcal{H}(\{a_2, a_4\})\}$

Intersection

Intersection And Satisfied Access Policy

- For the attributes set:
  - run $\mathcal{H}$ on *all non-empty subsets* of Υ
  - add all obtained elements to a set $\mathcal{X}$
  - run Eval on $\mathcal{X}$
- For the intersection:
  - Π is satisfied by Υ
  - $\iff \exists\, S \subseteq \Upsilon$ that satisfies one conjunction of Π
  - By construction, $\mathcal{H}(S) \in \mathcal{X}$ and $\mathcal{H}(S) \in \mathcal{Y}$
  - $\iff \mathsf{acc}_{\Upsilon} \cap \mathsf{accp}_{\Pi} = \{\mathcal{H}(S)\} \neq \varnothing$

An Example

- $\Pi = (a_1 \wedge a_3) \vee (a_2 \wedge a_4)$, $\mathcal{Y} = \{\mathcal{H}(\{a_1, a_3\}), \mathcal{H}(\{a_2, a_4\})\}$
- $\Upsilon = \{a_1, a_2, a_3\}$, $\mathcal{X} = \{\mathcal{H}(\{a_1\}), \mathcal{H}(\{a_2\}), \cdots, \mathcal{H}(\{a_1, a_2, a_3\})\}$
- $\mathcal{H}(\{a_1, a_3\}) \in \mathcal{X} \cap \mathcal{Y}$

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Asymmetric Bilinear Pairing Group

Cyclic (multiplicative) groups

Generator of $\mathbb{G}_2$

$$\Gamma = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$$

Prime number

Generator of $\mathbb{G}_1$

Asymmetric bilinear pairing

$e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ s.t.:

- $e(g_1^a, g_2^b) = e(g_1, g_2)^{ab}$ for all $a, b \in \mathbb{Z}_p$ **(bilinear)**
- $e(g_1, g_2) \neq 1$, 1: identity element of $\mathbb{G}_T$ (**non-degenerate**)
- $e(X, Y)$ **efficiently computable** for all $X \in \mathbb{G}_1, Y \in \mathbb{G}_2$

Asymmetric Bilinear Pairing Group

Cyclic (multiplicative) groups

Generator of $\mathbb{G}_2$

$$\Gamma = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$$

+ ***Efficient algorithm to decide membership of the groups***

Prime number

Generator of $\mathbb{G}_1$

Asymmetric bilinear pairing

$e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ s.t.:

- $e(g_1^a, g_2^b) = e(g_1, g_2)^{ab}$ for all $a, b \in \mathbb{Z}_p$ **(bilinear)**
- $e(g_1, g_2) \neq 1$, 1: identity element of $\mathbb{G}_T$ (**non-degenerate**)
- $e(X, Y)$ **efficiently computable** for all $X \in \mathbb{G}_1, Y \in \mathbb{G}_2$

Dual Pairing Vector Spaces (DPVS) [CLL+13]

Fixed constant dimension

$$\left(\ \Gamma, n,\ \mathbb{D} = (\boldsymbol{d}_1, \cdots, \boldsymbol{d}_n), \mathbb{D}^* = (\boldsymbol{d}_1^*, \cdots, \boldsymbol{d}_n^*)\ \right)$$

Asymmetric bilinear pairing group

Random bases of $\mathbb{Z}_p^n$

Dual Pairing Vector Spaces (DPVS) [CLL+13]

Fixed constant dimension

$$\left(\Gamma, n, \mathbb{D} = (\boldsymbol{d}_1, \cdots, \boldsymbol{d}_n), \mathbb{D}^* = (\boldsymbol{d}_1^*, \cdots, \boldsymbol{d}_n^*) \right)$$

Asymmetric bilinear pairing group

Random bases of $\mathbb{Z}_p^n$

Dual orthonormal, i.e. :
•$\boldsymbol{d}_i \cdot \boldsymbol{d}_j^* = 0$ whenever $i \neq j$
•$\boldsymbol{d}_i \cdot \boldsymbol{d}_i^* = \psi$ for all i, where $\psi \in \mathbb{Z}_p^*$

Dual Pairing Vector Spaces (DPVS) [CLL+13]

Fixed constant dimension

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Asymmetric bilinear pairing group

Random bases of $\mathbb{Z}_p^n$

$(\mathbb{D}, \mathbb{D}^*) \leftarrow \mathsf{Dual}(\mathbb{Z}_p^n)$

Dual orthonormal, i.e. :
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In our settings, $n = 2, \mathbb{D} = (\boldsymbol{d}_1, \boldsymbol{d}_2), \mathbb{D}^* = (\boldsymbol{d}_1^*, \boldsymbol{d}_2^*)$

Dual Pairing Vector Spaces (DPVS) [CLL+13]

Fixed constant dimension

$$\left(\Gamma, n, \mathbb{D} = (\boldsymbol{d}_1, \cdots, \boldsymbol{d}_n), \mathbb{D}^* = (\boldsymbol{d}_1^*, \cdots, \boldsymbol{d}_n^*) \right)$$

Asymmetric bilinear pairing group

Random bases of $\mathbb{Z}_p^n$

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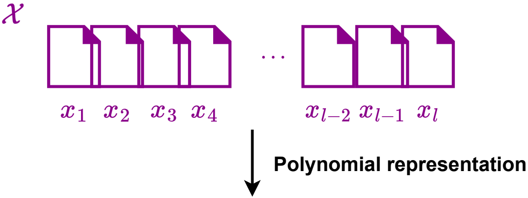
In our settings, $n = 2, \mathbb{D} = (\boldsymbol{d}_1, \boldsymbol{d}_2), \mathbb{D}^* = (\boldsymbol{d}_1^*, \boldsymbol{d}_2^*)$

All elements of $\mathbb{D}, \mathbb{D}^*$ are vectors!

Pairings and Vectors

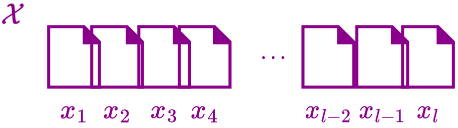
- $g_i \in \mathbb{G}_i$ group element for $i \in \{1, 2\}$, $\boldsymbol{u}, \boldsymbol{v}$ two vectors of length ℓ
- $g_i^{\boldsymbol{v}} := (g_i^{v_1}, \cdots, g_i^{v_\ell})$
- $g_i^{\boldsymbol{u} \cdot \boldsymbol{v}} = g_i^{\alpha}$, where $\alpha = \boldsymbol{u} \cdot \boldsymbol{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + \cdots + u_\ell \cdot v_\ell$
- $$e(g_1^{\boldsymbol{u}}, g_2^{\boldsymbol{v}}) := \prod_{i=1}^{\ell} e(g_1^{u_i}, g_2^{v_i}) = e(g_1, g_2)^{\boldsymbol{u} \cdot \boldsymbol{v}}$$

Characteristic Polynomial



$$Ch_{\mathcal{X}}[Z] = (x_1+Z)\cdot(x_2+Z)\cdots(x_l+Z) = \prod_{i=1}^{l}(x_i+Z) = \sum_{i=0}^{l} a_i Z^i$$

Characteristic Polynomial



$\downarrow$ **Polynomial representation**

$$Ch_{\mathcal{X}}[Z] = (x_1+Z)\cdot(x_2+Z)\cdots(x_l+Z) = \prod_{i=1}^{l}(x_i+Z) = \sum_{i=0}^{l} a_i Z^i$$

Evaluation at point s: $\mathrm{Ch}_{\mathcal{X}}(s) = \prod_{i=1}^{l}(x_i+s) = \sum_{i=0}^{l} a_i s^i$

Our Dually Computable Accumulator

First step: *private* evaluation and *public* witness generation

Our Dually Computable Accumulator

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Idea: using [Ngu05]'s pairing-based accumulator:

- $s \leftarrow \mathbb{Z}_p^*$
- $\Gamma = (p, \mathbb{G}, \mathbb{G}_T, g, e)$ **symmetric** pairing group
- $\mathsf{acc}_{\mathcal{X}} = g^{Ch_{\mathcal{X}}(s)}$
- $\mathsf{wit}_y = g^{Ch_{\mathcal{X} \setminus \{y\}}(s)}$
- *Verification:* $e(\mathsf{acc}_{\mathcal{X}}, g) \stackrel{?}{=} e(\mathsf{wit}_y, g^y \cdot g^s)$

Our Dually Computable Accumulator

First step: *private* evaluation and *public* witness generation

Idea: using [Ngu05]'s pairing-based accumulator:

- $\boxed{\mathsf{sk}_{\mathsf{acc}} = s \leftarrow \mathbb{Z}_p^*}$
- $\Gamma = (p, \mathbb{G}, \mathbb{G}_T, g, e)$ **symmetric** pairing group
- $\mathsf{acc}_{\mathcal{X}} = g^{Ch_{\mathcal{X}}(s)}$ privately computed
- $\mathsf{wit}_y = g^{Ch_{\mathcal{X} \setminus \{y\}}(s)}$ privately computed
- *Verification:* $e(\mathsf{acc}_{\mathcal{X}}, g) \stackrel{?}{=} e(\mathsf{wit}_y, g^y \cdot g^s)$ privately computed

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First step: *private* evaluation and *public* witness generation

Idea: using [Ngu05]'s pairing-based accumulator:

- $\mathsf{sk_{acc}} = s \leftarrow \mathbb{Z}_p^*$
- $\boxed{\mathsf{pk_{acc}} = (\mathit{\Gamma} = (p, \mathbb{G}, \mathbb{G}_T, g, e), g^s, g^{s^2}, \cdots, g^{s^q})}$ $\quad q \in \mathbb{N}$ **bound**
- $\mathsf{acc}_{\mathcal{X}} = g^{Ch_{\mathcal{X}}(s)} = g^{\sum_{i=0}^{q} a_i s^i} = \prod_{i=0}^{q} (g^{s^i})^{a_i}$ $\quad$ publicly computed
- $\mathsf{wit}_y = g^{Ch_{\mathcal{X}\setminus\{y\}}(s)} = g^{\sum_{i=0}^{q} b_i s^i} = \prod_{i=0}^{q} (g^{s^i})^{b_i}$ $\quad$ publicly computed
- *Verification:* $e(\mathsf{acc}_{\mathcal{X}}, g) \stackrel{?}{=} e(\mathsf{wit}_y, g^y \cdot g^s)$ $\quad$ publicly computed

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First step: *private* evaluation and *public* witness generation

Idea: using [Ngu05]'s pairing-based accumulator:

- $s \leftarrow \mathbb{Z}_p^*$
- $\Gamma = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$ **asymmetric** pairing group
- $\mathsf{acc}_{\mathcal{X}} = g_{\mathbf{1}}^{Ch_{\mathcal{X}}(s)}$
- $\mathsf{wit}_y = g_{\mathbf{2}}^{Ch_{\mathcal{X}\setminus\{y\}}(s)}$
- *Verification:* $e(\mathsf{acc}_{\mathcal{X}}, g_{\mathbf{2}}) \stackrel{?}{=} e(g_{\mathbf{1}}^{y} \cdot g_{\mathbf{1}}^{s}, \mathsf{wit}_y)$

Our Dually Computable Accumulator

First step: *private* evaluation and *public* witness generation

Idea: using [Ngu05]'s pairing-based accumulator:

- $\mathsf{sk}_{\mathsf{acc}} = s \leftarrow \mathbb{Z}_p^*$
- $\boxed{\mathsf{pk}_{\mathsf{acc}} = (\Gamma = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e), g_2^s, g_2^{s^2}, \cdots, g_2^{s^q})}$
- $\mathsf{acc}_{\mathcal{X}} = g_1^{Ch_{\mathcal{X}}(s)}$ privately computed
- $\mathsf{wit}_y = g_2^{Ch_{\mathcal{X}\setminus\{y\}}(s)} = g_2^{\sum_{i=0}^{q} b_i s^i} = \prod_{i=0}^{q} (g_2^{s^i})^{b_i}$ publicly computed
- *Verification:* $e(\mathsf{acc}_{\mathcal{X}}, g_2) \stackrel{?}{=} e(g_1^y \cdot g_1^s, \mathsf{wit}_y)$ **privately** computed

Our Dually Computable Accumulator

First step: *private* evaluation and *public* witness generation

Idea: using [Ngu05]'s pairing-based accumulator + DPVS of dimension 2

- $\mathsf{sk}_{\mathsf{acc}} = (s \leftarrow \mathbb{Z}_p^*, \boxed{(\mathbb{D}, \mathbb{D}^*) \leftarrow \mathsf{Dual}(\mathbb{Z}_p^2)})$
- $\boxed{\mathsf{pk}_{\mathsf{acc}} = (\Gamma, g_2^{\boldsymbol{d}_2^*}, g_2^{\boldsymbol{d}_2^* s}, g_2^{\boldsymbol{d}_2^* s^2}, \cdots, g_2^{\boldsymbol{d}_2^* s^q}, g_2^{\boldsymbol{d}_1^*}, g_1^{\boldsymbol{d}_2}, g_1^{\boldsymbol{d}_2 s})}$
- $\mathsf{acc}_{\mathcal{X}} = g_1^{\boldsymbol{d}_1 Ch_{\mathcal{X}}(s)}$ privately computed
- $\mathsf{wit}_y = g_2^{\boldsymbol{d}_2^* Ch_{\mathcal{X}\setminus\{y\}}(s)} = g_2^{\boldsymbol{d}_2^* \sum_{i=0}^{q} b_i s^i} = \prod_{i=0}^{q} (g_2^{\boldsymbol{d}_2^* s^i})^{b_i}$ publicly computed
- *Verification:* $e(\mathsf{acc}_{\mathcal{X}}, g_2^{\boldsymbol{d}_1^*}) \stackrel{?}{=} e(g_1^{\boldsymbol{d}_2 y} \cdot g_1^{\boldsymbol{d}_2 s}, \mathsf{wit}_y)$ publicly computed

Our Dually Computable Accumulator

Second step: *public* evaluation and *public* verification

- $\mathsf{sk}_{\mathsf{acc}} = (s \leftarrow \mathbb{Z}_p^*, (\mathbb{D}, \mathbb{D}^*) \leftarrow \mathsf{Dual}(\mathbb{Z}_p^2))$

- $\mathsf{pk}_{\mathsf{acc}} =$
$(\Gamma, g_2^{d_2^*}, g_2^{d_2^* s}, g_2^{d_2^* s^2}, \cdots, g_2^{d_2^* s^q}, g_2^{d_1^*}, g_1^{d_2}, g_1^{d_2 s}, \boxed{g_2^{d_1^* s}, \cdots, g_2^{d_1^* s^q}, g_1^{d_1}})$

- $\mathsf{accp}_{\mathcal{X}} = g_2^{d_1^* Ch_{\mathcal{X}}(s)} = g_2^{d_1^* \sum_{i=0}^{q} a_i s^i} = \prod_{i=0}^{q} (g_2^{d_1^* s^i})^{a_i}$ publicly computed

- *Verification:* $e(g_1^{d_1}, \mathsf{accp}_{\mathcal{X}}) \stackrel{?}{=} e(g_1^{d_2 y} \cdot g_1^{d_2 s}, \mathsf{wit}_y)$ publicly computed

All Properties Are Satisfied

- **Small sizes:** $|\mathsf{acc}| = 2 \cdot |\mathbb{G}_1|$, $|\mathsf{accp}| = 2 \cdot |\mathbb{G}_2|$, $|\mathsf{wit}| = 2 \cdot |\mathbb{G}_2|$

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$$\underbrace{e(\mathsf{acc}_{\mathcal{X}}, g_2^{\boldsymbol{d}_1^*})}_{\text{from Eval}} = \underbrace{e(g_1^{\boldsymbol{d}_2(y+s)}, \mathsf{wit}_y)}_{\text{from WitCreate}} = \underbrace{e(g_1^{\boldsymbol{d}_1}, \mathsf{accp}_{\mathcal{X}})}_{\text{from PublicEval}}$$

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- **Dual collision resistance**: from *q-Strong Bilinear Diffie Hellman* assumption, as Nguyen's scheme

Our CP-ABE Scheme

- Combination of previous ideas + our dually computable accumulator
- Protection against unauthorized decryption: relies on *characteristic polynomial property*
- Advantages:
  - Constant size ciphertext $(2 \cdot |\mathbb{G}_2|)$
  - Constant size secret key $(2 \cdot |\mathbb{G}_1|)$
- Drawbacks:
  - Public key size exponential in the number of attributes in the scheme
  - No generic construction and No security reduction
  - Simple access policies

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Other Main Contribution

- New pairing-based Key Policy Attribute-Based Encryption with both *constant size ciphertext and secret keys* based on Cryptographic Accumulators

Other Auxiliary Contribution

- First (pairing-based) Cryptographic Accumulator scheme with *private evaluation* and *public witness generation*

All results are in an article accepted at CANS 2023

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Conclusion

- Aim of this Phd thesis: **building efficient and secure schemes for data sharing**
- How did we do this? By **establishing relations between primitives**
  - Link between Broadcast Encryption and Identity-Based Encryption with Wildcards
  - Link between Attribute-Based Encryption and Cryptographic Accumulators
- Means: introducing **new properties and functionalities** for building block primitives

Summary of Our Works

Contribution	In submission	Accepted
Broadcast Encryption from WIBE		CANS 2022
ABE from Accumulators		CANS 2023
ABE from WIBE	$\checkmark$	
SoK on Accumulators	$\checkmark$	

Future Works

Improving current results

- Create a constant size ciphertext pattern-hiding Identity-Based Encryption with Wildcards scheme
- Develop a generic construction of ABE from Dually Computable Cryptographic Accumulators
- Reduce our CP-ABE public key size and deal with fine-grained access policies

Going further

- Develop quantum resistant schemes
- Study the relation between Cryptographic Accumulators and another recently introduced primitive, Locally Verifiable Aggregate Signature[a]

[a] Short article about it accepted at CFAIL 2023

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